12d Model

Civil and Surveying Software

Drainage Analysis Module

Hydraulics

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This document defines the various terms relevant to the hydraulic calculations performed by the *Drainage Analysis* module of *12d Model*, including how and where they are calculated.

The terms *pipe* and *channel* are sometimes used interchangeably, as are the terms *pit*, *manhole*, *inlet* and *structure*.

Definitions:

Q_{rat} = Peak Flow
The peak flow rate for the pipe, without consideration of bypass flows. It is determined from the contributing catchments via the Rational Method (and includes any *direct* pit or pipe flow that may have been specified upstream). It is shown in the Hydraulic Report as "Peak Flow Qrat", and is also set as a pipe attribute named "calculated peak flow".

Q_b = Net Bypass Flow
When considering bypass flows, Q_b for the pipe is determined from an analysis of *all* the upstream pits contributing to the pipe, as recommended in the *Australian Rainfall and Runoff*. It represents the sum of the peak bypass flows *approaching* these pits, *minus* the sum of the peak bypass flows *leaving* them. It is shown in the Hydraulic Report as "Net Bypass Flow Qb", and is also set as a pipe attribute named "calculated net bypass flow". **Note:** the peak bypass flow leaving each pit, is determined from the pit inlet capacity data, defined for each pit type in the *drainage.4d* setup file, as well as the relevant choke factor determined for each pit.

Q = Pipe Flow = $Q_{rat} + Q_b$
The peak flow rate in the pipe, represented by the sum of Q_{rat} and Q_b. It is shown in the Hydraulic Report as "Pipe Flow Q", and is also set as a pipe attribute named "pipe flow". **Note:** it is only when considering bypass flows, that Q may differ from Q_{rat}.

Q_x = Excess Pipe Flow
If the pipe flow, Q, is so large that the HGL would rise *above* the upstream pit's grate level, then Q_x will represent the difference between Q and the flow rate that would keep the HGL *just* at the grate level. It is shown in the Hydraulic Report as "Excess Pipe Flow Qx", and is also set as a pipe attribute named "calculated excess flow". **Note:** the excess flow amount, Q_x, is still included in the pipe flow, Q, and *is not* automatically re-routed to the overland system. However, when considering bypass flows, the user may set the *Qx routing increment* to a value greater than zero, which will initiate extra analysis passes to re-route just enough flow as is necessary, from the pipe network to the overland system, so as to eradicate the excess pipe flows. At each extra pass, an amount no greater than the *Qx routing increment* is removed from the pipe network – from the most upstream pipe(s) with excess flow(s) – and re-routed to the overland system. If flow is entering from the top of a pipe's upstream pit, the re-routing is achieved initially by reducing the pit's choke factor by an amount calculated to have the same effect. Once the choke factor is reduced to zero, however, any remaining Q_x is re-routed as *pit surcharge flow*, Q_s.

Q_s = Pit Surcharge Flow
When (and only when) excess pipe flows are re-routed to the overland system, it is possible for some pits to surcharge (flow rising up and exiting the top of the pit). At such pits, the choke factor will have been reduced to zero, and Q_s will have a *positive* value equal in magnitude to the *negative* pit inflow. In the Hydrology Report, a negative value of "Inlet Flow Qi" indicates a surcharge flow, and Q_s is also set as a pit attribute named "calculated surcharge flow". **Note:** pit surcharge flow is automatically included in the bypass flow leaving a pit, and *may* re-enter the pipe network elsewhere.

Definitions: *cont …*

V_f = Full Pipe Velocity = Q / A_f
The velocity in the pipe when the pipe flow, Q, fills the entire cross-sectional area, A_f, of the pipe. This is the minimum velocity possible for this flow rate. It is shown in the Hydraulic Report as "Full Pipe Vel Vf=Q/Af", and is also set as a pipe attribute named "full pipe velocity". **Note:** V_f is the velocity used to determine pressure head and water surface elevation losses through the upstream pit. Typically, the loss coefficients K_u & K_w only apply to pipes under pressure.

Q_{cap} = Capacity Flow
For the particular size, grade and roughness of the pipe, and assuming no downstream tailwater restrictions, the capacity flow is the flow rate theoretically possible in the pipe, at the point where the flow would become pressurised (i.e. flowing *exactly* full). It is shown in the Hydraulic Report as "Capacity Flow Qcap", and is also set as a pipe attribute named "flow capacity".

V_{cap} = Capacity Velocity = Q_{cap} / A_f
The velocity corresponding to Q_{cap}. It is shown in the Hydraulic Report as "Capacity Vel Vcap=Qcap/Af", and is also set as a pipe attribute named "capacity velocity". **Note:** V_{cap} is sometimes referred to by others as *at grade velocity*, or more confusingly, as *full pipe velocity*. Great care should be taken, to avoid confusing V_{cap} with V_f.

Capacity Ratio = Q / Q_{cap}
This is simply the ratio of the pipe flow to the capacity flow. If greater than 1.0, Q will always be pressurised. If less than 1.0, Q may or may not be pressurised, depending on the conditions downstream. Some governing authorities specify that it should always be less than 1.0. It is shown in the Hydraulic Report as "Q/Qcap Ratio", and is also set as a pipe attribute named "flow capacity ratio".

Q_{mcap} = Max Capacity Flow
Similar to Q_{cap}, but representing the maximum unpressurised flow rate theoretically possible in the pipe. For a circular pipe, Q_{mcap} occurs at a depth of about 0.94xD. For a box-culvert, Q_{mcap} occurs at a depth just below the obvert (0.999xH, say). For circular pipes, Q_{mcap} is approximately 1.07xQ_{cap}. For box-culverts, Q_{mcap} may be considerably greater than Q_{cap}, due to the sudden increase in friction losses when the flow comes into contact with the top wall – an effect more pronounced as the culvert's width/height ratio increases. **Note:** Q_{mcap} is not calculated by 12d Model, as it is an inherently unstable flow, which can become pressurised with only the slightest increase in frictional resistance. Pipe manufacturers typically publish capacities which match more closely to Q_{cap}.

V_{mcap} = Max Capacity Velocity
The velocity corresponding to Q_{mcap}, that is: $V_{mcap} = Q_{mcap} / A_w$, where A_w is the wetted cross-sectional area at the depth at which Q_{mcap} occurs.

Definitions: *cont …*

d_n = Normal Depth
The depth in the pipe when the slope of the water surface of the pipe flow, Q, is parallel to the pipe slope. If Q is greater than Q_{cap}, d_n is set to the obvert of the pipe. It is set as a pipe attribute named "normal depth", and the value [d_n / <pipe height>] is set as "normal depth relative".

d_c = Critical Depth
The depth in the pipe when the local energy head (the sum of the pressure and velocity heads only, ignoring the gravity head) of the pipe flow, Q, is at a minimum. It is set as a pipe attribute named "critical depth", and the value [d_c / <pipe height>] is set as "critical depth relative".

V_n = Normal Depth Velocity
The velocity in the pipe when the pipe flow, Q, is flowing at the normal depth, d_n. This is the maximum velocity possible for this flow rate, in the most common case of a "steep" slope pipe (where $d_c > d_n$). In the rare case of a "mild" slope pipe (where $d_c < d_n$), it is possible for the velocity to be slightly greater near the downstream end of the pipe, if the flow "drops off the end" and the depth approaches d_c. It is shown in the Hydraulic Report as "Norm Depth Vel Vn=Q/An", and is also set as a pipe attribute named "normal velocity".

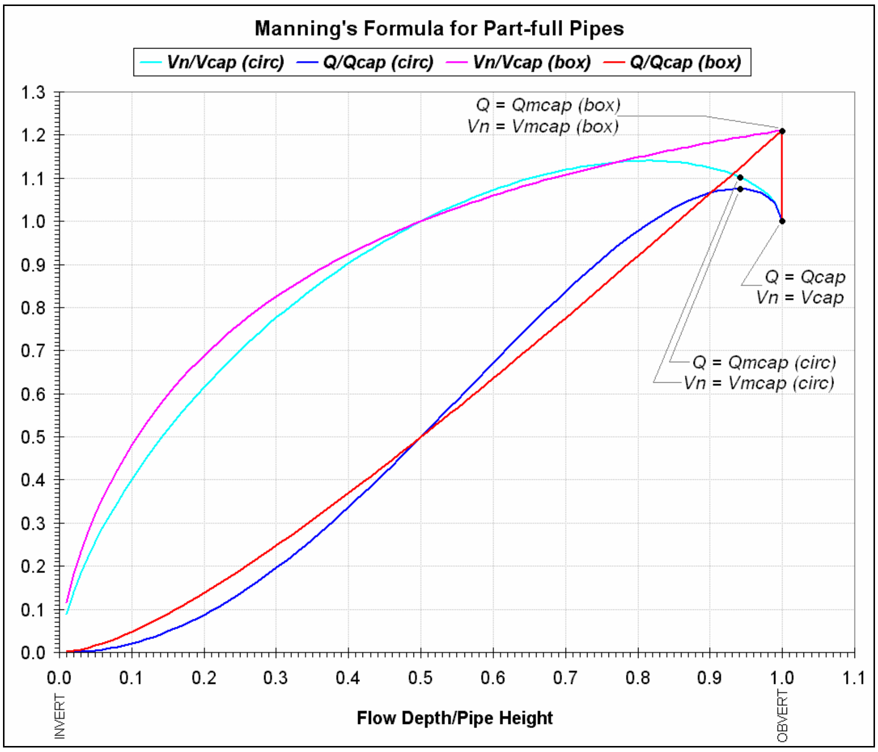
V_c = Critical Depth Velocity
The velocity in the pipe when the pipe flow, Q, is flowing at the critical depth, d_c. When the flow is slower than V_c (deeper than d_c) it is termed *tranquil flow*. When the flow is faster than V_c (shallower than d_c), it is termed *rapid flow*. It is shown in the Hydraulic Report as "Crit Depth Vel Vc=Q/Ac", and is also set as a pipe attribute named "critical velocity".

V_{min} & V_{max} = Allowable Velocity Ranges for V_n and V_{cap}
These velocity limits are often specified by governing authorities, to ensure that all pipe flows are fast enough to wash away impediments, but not so fast as to scour the pipes. If set on the *GLOBAL=>Main* tab of the *Drainage Network Editor*, they are checked against the calculated V_n and/or V_{cap} values. If ever a calculated velocity is too low or too high, a new pipe grade is recommended in the *Output Window*, to help remedy the problem.

V_a = Actual Velocity (VicRoads manual)
The Victorian "VicRoads" manual refers to a chart for determining the so-called *actual velocity*, V_a, in part-full pipes. The chart shows a relationship between Q / Q_{cap} values and V_a / V_{cap} values. The resultant V_a values from this chart, are equivalent to the normal depth velocity, V_n, values calculated by 12d Model.

Manning's Formula versus the Colebrook-White Formula:

The *Drainage Analysis* module allows pipe roughness to be specified as either Manning's *n* (based on metre-second units), or Colebrook's *k* (specified in mm in 12d). The choice of roughness type determines the formula – Manning or Colebrook-White – used to calculate: Q_{cap}, V_{cap}, d_n and V_n. It also governs how the friction slope is determined in both the Darcy-Weisbach equation and the Gradually Varied Flow equation, when calculating the HGL values along the pipe. The Colebrook-White formula may be seen as an empirical combination of the Rough and Smooth Pipe Laws of turbulent flow, which were both derived from boundary-layer theory by Prandtl and von Kármán, but adjusted to match the experimental results of Nikuradse, who performed a series of lab tests on small pipes of uniform roughness. Colebrook and White, however, performed tests on a range of *commercial* pipes (of non-uniform roughness) – with an equivalent *k* deduced from the constant friction factor, *f*, observed at high Reynolds number – and found that for most of the pipes they tested, their new combination-formula matched well. The turbulent range of the now widely-used *Moody Diagram*, is wholly based on the Colebrook-White formula, and represents the trend to be expected, in the absence of any specific data for particular pipes. Where non-circular pipes are used, or where part-full flow occurs in any kind of pipe or open channel of reasonably uniform cross-section, substitution of 4*R* for *D* (four times the *hydraulic radius* for diameter) can be justified, with the assumption that the mean shear-stress around the wetted perimeter is not too dissimilar from the uniform stress around a circular perimeter (true enough for box-culverts and many open channels). Because the Colebrook-White formula accounts for the variations in *f* dependent on the *relative* roughness (*k*/*R* or *k*/*D*) and the Reynolds number (Re), it is *slightly* more reliable, when considered across the range of pipe sizes and flow rates commonly found in stormwater systems. Most of the values published for Manning’s *n* are for relatively large open channels, where the dependency on Re is slight, and relative roughness is implied in the *n* value. As such, Manning’s formula is good for larger systems (especially natural channels), but when applied to smaller systems like typical stormwater pipes, the Colebrook-White formula suggests that these published *n* values should be reduced somewhat. Overall, the primary difficulty with *both* formulae lies in the selection of suitable *n* or *k* values, and significant errors are not uncommon. The following figure shows a normalised graph of Manning's formula, for a full range of flow depths in both circular pipes and (square) box-culverts. (The corresponding Colebrook-White graph is so similar on this normalised scale, that it may be considered identical.) Note the difference between Q_{cap} and Q_{mcap} on the graph.

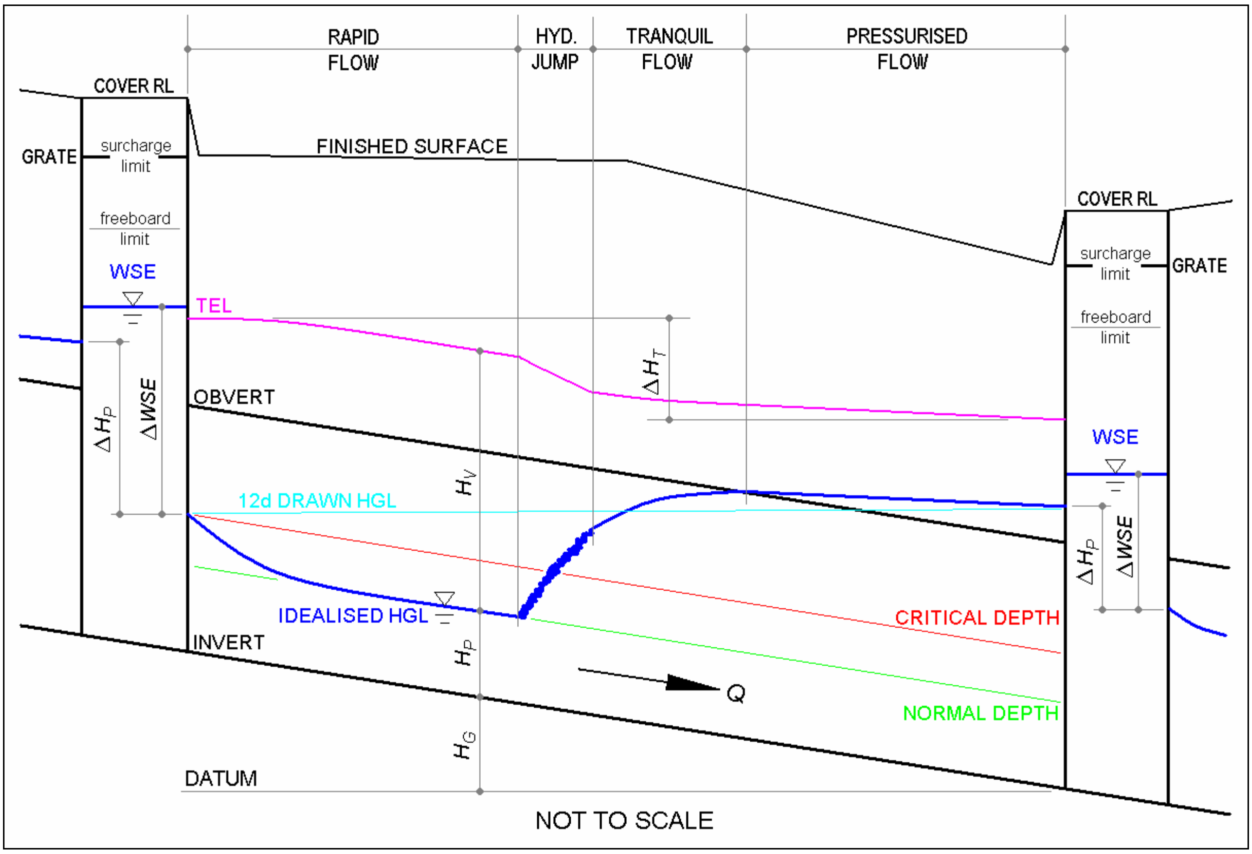


The Hydraulic Grade Line (HGL):

In 12d Model, when a Drainage string is profiled or plotted, you have the option to display the HGL determined from a storm analysis of the drainage network. The HGL represents the peak sum of the pressure and gravity heads throughout the network, and in 12d, it is idealised to show separate head losses along the pipes and through the pits. *In reality, these losses are not separate, but continuous, with the often highly un-developed flow near the pits, progressively forming fully-developed, one-dimensional flow, in the pipe reaches sufficiently distant from the pits.*

HGL Along Pipes: The HGL drawn along a pipe in 12d, is simply the straight line joining the pipe's idealised entrance and exit HGL levels. As such, in pipes where the steady flow is wholly pressurised, it is a good representation of the friction slope, and consequent loss of total head (ΔH_T) due to friction. However, where part-full flow occurs in any portion of the pipe, neither the friction slope nor the water surface slope is constant, and so the straight line implies nothing other than the idealised entrance and exit HGL levels. For these cases, the idealised HGL is calculated internally, assuming fully-developed flow, via numerical integration of the Gradually Varied Flow equation. This handles all possible cases of tranquil and rapid flow on mild and steep slope pipes, including the hydraulic jumps that can occur on steep slopes. Note that, due to the non-uniform velocity head in these cases, ΔH_T due to friction cannot be determined from the idealised HGL, and must instead be determined from the Total Energy Line (TEL).

HGL At Pits: The horizontal HGL drawn across a pit, represents the peak water surface elevation (WSE) in that pit, and is tested against the freeboard and surcharge limits imposed there. The jump in the HGL between a pit's entrance and exit, represents the change (normally a loss) in pressure head (ΔH_P) through that pit (it does *not* represent the loss in total head). If a pit's entrance HGL level is higher than the level formed by the minimum of d_n and d_c in an upstream pipe, then this HGL level also forms a tailwater condition for that upstream pipe. Note that ΔWSE is typically equal to, or slightly greater than ΔH_P, and that both these jumps may, to a certain extent, be thought of as the effect that a pit has on what would otherwise be fully-developed, one-dimensional flow.



Schematic of a typical HGL in a steep slope pipe

Hydraulic Equations:

Bernoulli's Equation (for the head of a streamline)

$$H_T = H_V + H_P + H_G = \frac{V^2}{2g} + h + z$$

Reynolds Number (laminar flow < 2000 < critical zone < 4000 < turbulent flow)

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{4VR}{\nu}$$

Darcy-Weisbach Equation (for steady, uniform [pressurised] flow)

$$\frac{-\Delta H_T}{L} = S_f = \frac{f}{D}\frac{V^2}{2g} = \frac{f}{4R}\frac{V^2}{2g} = \frac{m}{R}\frac{V^2}{2g}$$

Gradually Varied Flow Equation (for steady, non-uniform [free-surface] flow)

$$\frac{dh}{dL} = \frac{S_0 - S_f}{\left(1 - \dfrac{V^2 B}{g A_w}\right)}$$

Manning's Formula (for complete turbulence, high Re)

$$\sqrt{\frac{8g}{f}} = \sqrt{\frac{2g}{m}} = \frac{R^{1/6}}{n}$$ (Manning's relation to Chézy's coefficient, for metre-second units)

$$V = \frac{S^{1/2} R^{2/3}}{n}$$ (for metre-second units) **OR** $$V = 1.49\frac{S^{1/2} R^{2/3}}{n}$$ (for foot-second units)

Colebrook-White Formula (for transition zone flow and complete turbulence, Re > 4000)

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$ (as published by Colebrook, 1939)

$$V = -2\sqrt{2gDS}\,\log_{10}\left(\frac{k}{3.7D} + \frac{2.51\nu}{D\sqrt{2gDS}}\right) = -4\sqrt{2gRS}\,\log_{10}\left(\frac{k}{14.8R} + \frac{0.314\nu}{R\sqrt{2gRS}}\right)$$

Pressure Head Change Formula (through pit directly upstream of pipe)

$$-\Delta H_P = K_u V_f^{\,2}/2g$$

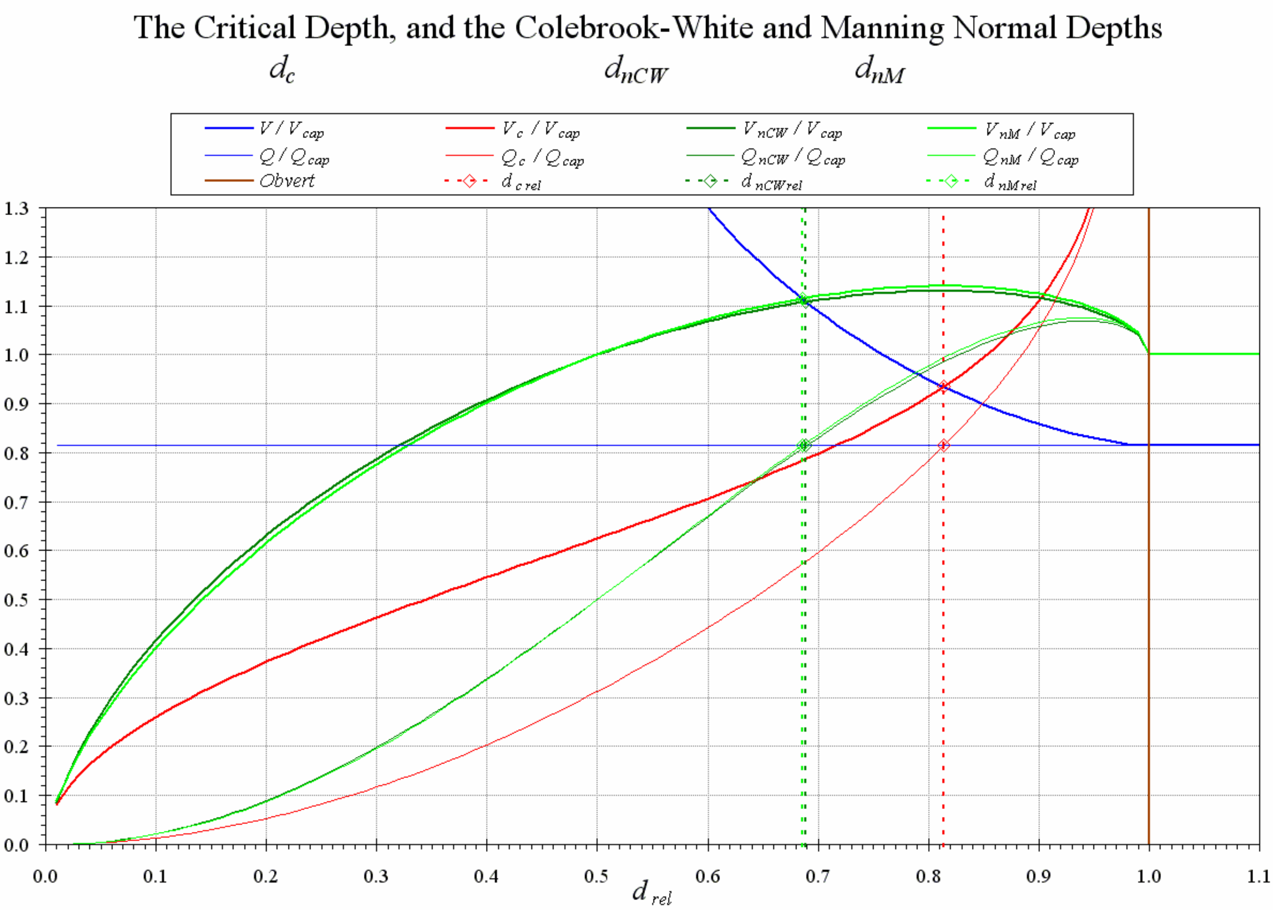
WSE Change Formula (between pit directly upstream of pipe and pipe entrance)

$$-\Delta WSE = K_w V_f^{\,2}/2g$$

where:		**[dimensions]**
H_T	= total head (energy per unit weight of water)	[L]
H_V	= velocity head = $V^2 / 2g$	[L]
H_P	= pressure head = h	[L]
H_G	= gravity head = z	[L]
V	= mean velocity of flow through a cross-section	[L/T]
g	= acceleration due to gravity	[L/T^2]
h	= pressure head (and depth of flow in an open channel) at a cross-section	[L]
z	= height of channel invert at a cross-section, from a constant horizontal datum	[L]
Re	= Reynolds number	[-]
ν	= kinematic viscosity of water	[L^2/T]
D	= diameter of circular pipe	[L]
R	= hydraulic radius of flow at a cross-section = $A_w / P_w = D/4$ for full circular pipe	[L]
A_w	= wetted cross-sectional area	[L^2]
P_w	= wetted cross-sectional perimeter	[L]
L	= plan length of channel	[L]
f	= Darcy friction factor (based on D)	[-]
m	= Fanning friction factor (based on R) = $f/4$ … confusingly, often denoted by f	[-]
dh/dL	= longitudinal slope of water surface at a cross-section, relative to S_0	[-]
B	= lateral breadth of water surface at a cross-section	[L]
S_0	= channel slope	[-]
S_f	= friction slope (energy lost, per unit weight of water, per unit length of channel)	[-]
S	= context dependent – typically: S_0 when solving for V; S_f when solving for S	[-]
n	= Manning roughness factor	[T/L$^{1/3}$]
k	= Colebrook roughness factor	[L]
WSE	= water surface elevation in pit	[L]
K_u	= pressure head change factor for pit directly upstream of pipe	[-]
K_w	= WSE change factor for pit directly upstream of pipe	[-]
V_f	= full pipe velocity	[L/T]

Determining the Critical and Normal Depths in Part-full Pipes:

Wherever the pipe flow, Q, is less than the capacity flow, Q_{cap}, the potential exists for part-full (free surface) flow to occur. In these instances, the critical and normal flow depths are determined by 12d Model, using the equations shown in the figure below. Note that the graph within the figure, shows the solutions for a circular pipe on a normalised scale.



$$Q = const < Q_{cap}$$

$$V_{cap} = -4\sqrt{2gRS_0}\,\log_{10}\left(\frac{k}{14.8R} + \frac{0.314\nu}{R\sqrt{2gRS_0}}\right) = \frac{S_0^{1/2} R^{2/3}}{n}$$

$$Q_{cap} = AV_{cap}$$

$$V = \frac{Q}{A_w}$$

$$V_c = \sqrt{\frac{gA_w}{B}}$$

$$V_{nCW} = -4\sqrt{2gR_wS_0}\,\log_{10}\left(\frac{k}{14.8R_w} + \frac{0.314\nu}{R_w\sqrt{2gR_wS_0}}\right)$$

$$V_{nM} = \frac{S_0^{1/2} R_w^{2/3}}{n}$$

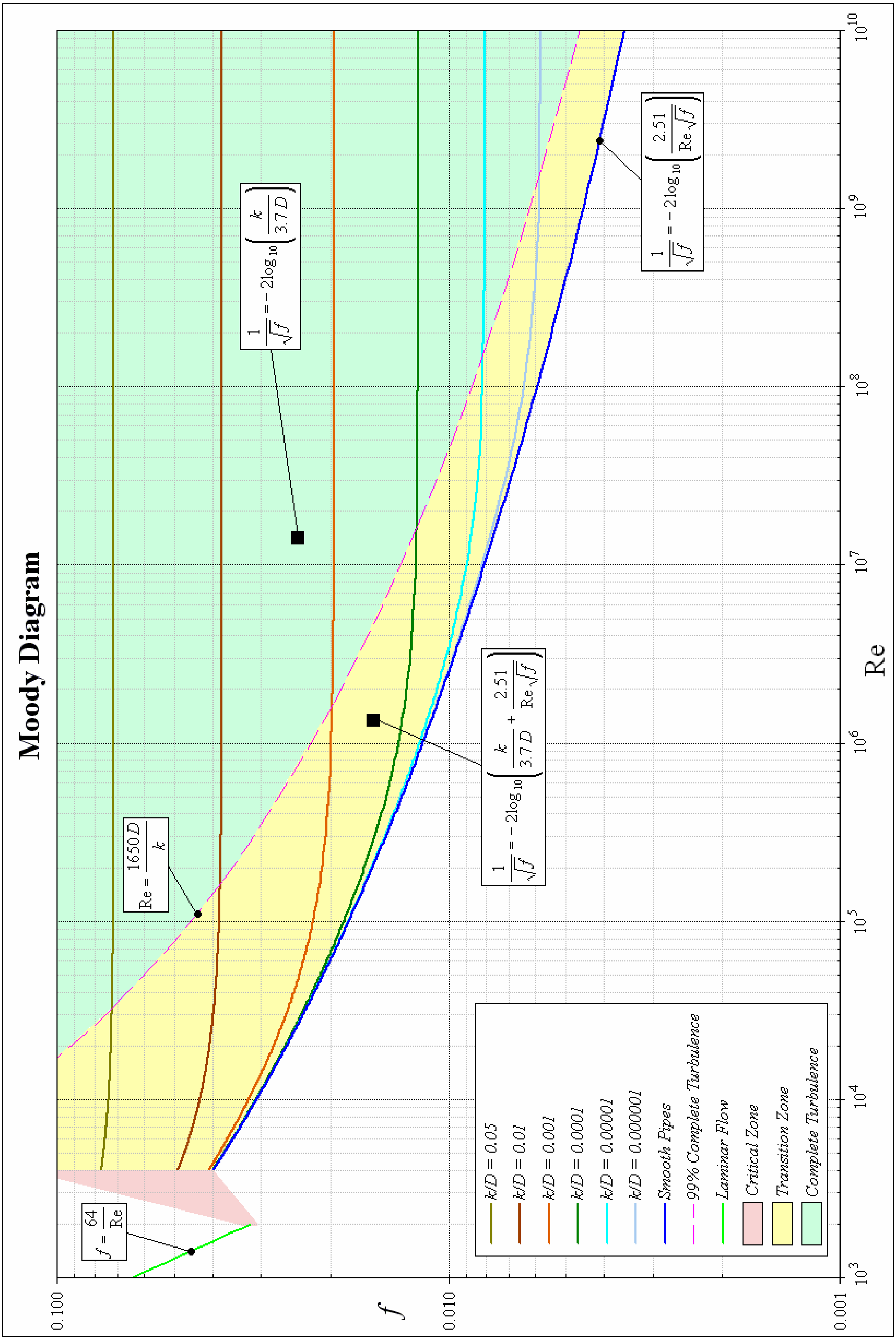
$$Q_c = A_wV_c; \quad Q_{nCW} = A_wV_{nCW}; \quad Q_{nM} = A_wV_{nM}$$

Circular Pipes	**Box Culverts**
$A = \pi D^2 / 4$	$A = WH$
$R = D/4$	$R = WH/(2W + 2H)$
$\theta = \cos^{-1}(1 - 2d_{rel})$	
$B = D\sin\theta$	$B = W$
$A_w = (\theta - \tfrac{1}{2}\sin(2\theta))D^2/4$	$A_w = WH\,d_{rel}$
$R_w = A_w/(D\theta)$	$R_w = A_w/(W + 2H\,d_{rel})$

$$d_{c\,rel} = d_{rel}\,|(V = V_c) = d_{rel}\,|(Q = Q_c)$$

$$d_{nCW\,rel} = d_{rel}\,|(V = V_{nCW}) = d_{rel}\,|(Q = Q_{nCW})$$

$$d_{nM\,rel} = d_{rel}\,|(V = V_{nM}) = d_{rel}\,|(Q = Q_{nM})$$


Moody Diagram
Re
f
0.100
0.010
0.001
10^3
10^4
10^5
10^6
10^7
10^8
10^9
10^{10}
$f = \frac{64}{Re}$
$Re = \frac{1650D}{k}$
$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k}{3.7D}\right)$
$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right)$
$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{2.51}{Re\sqrt{f}}\right)$
k/D = 0.05
k/D = 0.01
k/D = 0.001
k/D = 0.0001
k/D = 0.00001
k/D = 0.000001
Smooth Pipes
99% Complete Turbulence
Laminar Flow
Critical Zone
Transition Zone
Complete Turbulence

More Hydraulic Equations (for reference only – not used in 12d):

Chézy's Formula (for complete turbulence, high Re)

$$V = \sqrt{\frac{8gRS}{f}} = C\sqrt{RS} \qquad (C \text{ is Chézy's coefficient})$$

Laminar Flow Law (for Re < 2000)

$$f = \frac{64}{\mathrm{Re}} \qquad \text{(via Poiseuille's formula)}$$

Smooth Pipe Law (for Re > 4000, very low *k/D*)

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \qquad \text{(adjusted to Nikuradse's data from uniform roughness pipes)}$$

$$\approx -1.8\log_{10}\left(\frac{7}{\mathrm{Re}}\right) \qquad \text{(Colebrook's approximation: } f \text{ within } \pm 1.5\% \text{ for } 5000 < \mathrm{Re} < 10^8\text{)}$$

$$\approx -2\log_{10}\left(\frac{5.76}{\mathrm{Re}^{0.9}}\right)$$

Rough Pipe Law (for complete turbulence, high Re)

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{k}{3.7D}\right) \qquad \text{(adjusted to Nikuradse's data from uniform roughness pipes)}$$

$$\mathrm{Re} > \frac{170}{\sqrt{f}}\frac{D}{k} \qquad \text{(range of Rough Pipe Law, based on Nikuradse's observations)}$$

$$\mathrm{Re} > \frac{1650D}{k} \qquad \text{(range of Rough Pipe Law, based on C-W formula, for } f \text{ accurate to } < 1\%\text{)}$$

Converting *k* to *n* (by equating Manning's formula to the Colebrook-White formula)

$$n = \frac{R^{1/6}}{-4\sqrt{2g}\,\log_{10}\left(\frac{k}{14.8R} + \frac{0.314\nu}{R\sqrt{2gRS}}\right)}$$

Sources:

Colebrook, C.F., 1939, "Turbulent Flow in Pipes with particular reference to the Transition Region between the Smooth and Rough Pipe Laws", *J. Instn. Civ. Engrs.*, Vol. 11, 133-156.

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Pilgrim, D.H. (Ed.), 1987, "Australian Rainfall and Runoff ", Vol. 1, *Instn. Engrs. Aust.*